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Application of the Bearing Trace Hough Transform (BTHT) to Passive Shipping Lane Monitoring

HAW-JYE SHYU

*Advanced Information Technology Branch
Information Technology Division*

YUNG P. LEE

*Science Applications International Corp.
McLean, VA*

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13. ABSTRACT (Maximum 200 words) A modified Hough transform to detect targets on sonar B-scan produced from data collected by a passive acoustic sensor array is presented. Assuming that the target maintains a constant course and speed during the observation period, the target trace on the passive sonar B-scan can be shown as an arctangent function defined by the following target track parameters: speed, course, range at the closest point of approach (CPA), and CPA time. Detection of the target traces on the sonar B-scan can then be formulated as detection of the arctangent curves using the modified Hough transform. The track parameters of the target can be estimated and used to identify different targets.				
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APPLICATION OF THE BEARING TRACE HOUGH TRANSFORM (BTHT) TO PASSIVE SHIPPING LANE MONITORING

1. INTRODUCTION

When held by a passive sensor array, a moving target generates a dark trace on the sonar B-scan of the sensor array. Unlike the broadband correlogram, which is a two-dimensional (2-D) display of broadband acoustic energy in the delay-time vs time space, the passive sonar B-scan is a 2-D display of the narrowband acoustic energy distribution in the beam-angle vs time space. The horizontal coordinate of sonar B-scan indicates beam angle (i.e., target's bearing relative to the sensor array), while the vertical coordinate indicates the observation time, and the pixel value represents the level of the acoustic energy. It is produced with data collected by the sensor array using either a conventional or an adaptive beamformer. As a target moves through a surveillance area, the distribution of the acoustic energy seen by the sensor array changes. The acoustic energy peak will move across the beams as the target moves across the surveillance area. Over an observation period, the loci of the acoustic energy of the target form a dark trace on the sonar B-scan.

Figure 1 is an example of a sonar B-scan. The horizontal axis is the array look direction (or bearing of the target) from 45° to 135° . Here 90° corresponds to the broadside of the array while 0° and 180° correspond to endfires of the array. The vertical axis is the observation time of 128 minutes. Each visible dark trace in Fig. 1 corresponds to a source — each trace has its own set of track parameters. We call these dark traces the bearing traces (or beam curves) because each trace shows the changes in bearing of the corresponding target over the observation period. Some of these bearing traces have rapid bearing changes corresponding to fast-moving targets or targets at close range; some of the bearing traces are darker than others, indicating a higher level of acoustic energy.

The bearing trace generated by a target traveling with a constant speed and course has the form of an arctangent function defined by the target track parameters: speed, course, range at the closest point of approach (CPA), and CPA time. Based on this arctangent function, a modified Hough transform — the Bearing Trace Hough Transform (BTHT) — has been developed to detect target traces on the sonar B-scan. The track parameters of the detected arctangent curves can be used to separate different targets and provide useful information for passive shipping lane monitoring. Although different variations of the Hough transform [1-8] have been proposed to detect different analytic curves in different type of images, to the authors' knowledge, this is the first attempt to use the modified Hough transform to detect bearing traces of the sonar B-scan.

This report is organized as follows. Section 2 shows the derivation of the arctangent function for the bearing trace. Section 3 discusses the BTHT. Section 4 discusses the application of the BTHT to a shipping lane monitoring system. Section 5 shows the response of the BTHT for a special case under ideal conditions. Section 6 shows results of applying the BTHT to a sonar B-scan generated by a conventional plane-wave beamformer. Section 7 presents conclusions and discussion.

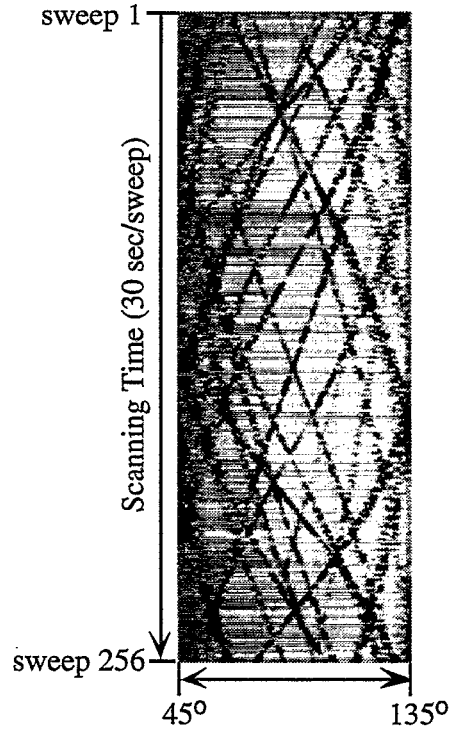


Fig. 1 — Sonar B-scan generated by a conventional plane-wave beamformer

2. THE ANALYTIC EQUATION OF THE BEARING TRACE

The derivation of the analytic equation for the bearing trace is based on the target and sensor geometry shown in Fig. 2. The center of the sensor array is located at the origin of the XY-coordinate system. The target is traveling with a constant speed V along the course θ marked as the thick dark line, the target range is R at the CPA point, and the time of the CPA is t_{cpa} . At the CPA, the target coordinates are given by

$$x_{cpa} = -R \sin \theta$$

and

$$y_{cpa} = R \cos \theta.$$

From Fig. 2, the target coordinates at time t can be written as

$$x(t) = x_{cpa} + (t - t_{cpa})V \cos \theta$$

and

$$y(t) = y_{cpa} + (t - t_{cpa})V \sin \theta.$$

The bearing of the target relative to the sensor array is defined to be

$$B(t) \equiv |atan2(y(t), x(t))|,$$

where $atan2(y, x)$ is the principal phase of the complex number $x+iy$. This arctangent's range is $-\pi < atan2(y, x) \leq \pi$, with

$$\begin{aligned} atan2(y, x) &> 0 \text{ if } y > 0; \\ atan2(y, x) &< 0 \text{ if } y < 0; \\ atan2(y, x) &= 0 \text{ if } y = 0, x \geq 0; \\ atan2(y, x) &= \pi \text{ if } y = 0, x < 0. \end{aligned}$$

Note that $0 \leq |atan2(y, x)| \leq \pi$, where $|\cdot|$ is the absolute value function. After simple manipulation, the bearing of the target can be written as

$$B(t) = \left| atan2 \left(\frac{R}{V} \cos \theta + (t - t_{cpa}) \sin \theta, -\frac{R}{V} \sin \theta + (t - t_{cpa}) \cos \theta \right) \right|. \quad (1)$$

Both the target course and the beam angle are measured counter-clockwise from the positive X-axis. The bearing ranges from 0° to 180° while the target course ranges from 0° to 360° . This is because a linear sensor array cannot differentiate two tracks when one is a reflection of the other from the sensor array. Over the observation period, the target bearing will form a trace on the sonar B-scan according to Eq. (1). Note that R and V only occur as the ratio R/V . Thus, as long as R/V remains the same, many different sets of R and V will generate the same bearing trace. This arctangent function facilitates the application of a modified Hough transform (i.e., the BTHT) to detect the target bearing trace on the narrowband sonar B-scan.

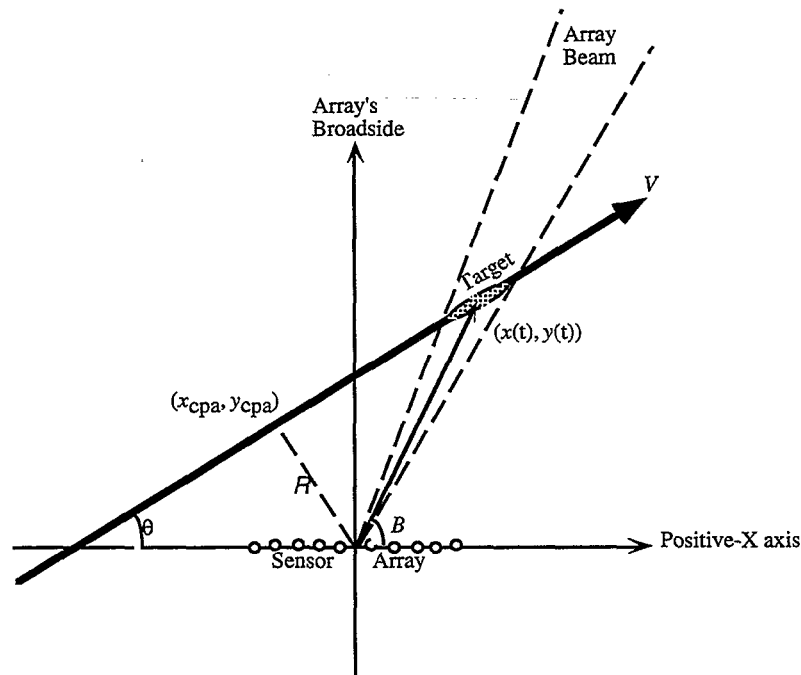


Fig. 2 — Target track relative to the sensor array

3. THE BEARING TRACE HOUGH TRANSFORM

As pointed out by Stockman and Agrawala [9], the Hough transform can be viewed either as a backprojection of pixels from the image space to the parameter space (i.e., a *distributed scheme*) or as a template-matching technique that integrates pixel values in the image space according to the template generated by the accumulator cell (i.e., an *integrated scheme*). The implementation of the BTHT presented in this report is based on the integrated scheme of the Hough transform. For each accumulator cell, a bearing trace template is generated according to the arctangent function shown in Eq. (1). The pixel values are then integrated along the bearing trace and normalized by the total number of pixels in the bearing trace. This normalized value is stored in the corresponding accumulator cell. The BTHT can be written as

$$f\left(\frac{R}{V}, \theta, t_{\text{cpa}}\right) = \frac{1}{N} \iint F(x, t) \delta(x - B(t)) dx dt \quad (2)$$

where

f is the output of BTHT;

N is the number of pixels involved in the integration;

t is the scan time;

x is the bearing angle;

$F(x, t)$ is the sonar B-scan pixel value at (x, t) ;

$B(t)$ is the bearing of the target at time t ;

$\delta(\cdot)$ is the Dirac-delta function indicating that the integration of $F(x, t)$ should be performed along the curve specified by the bearing trace $B(t)$.

As indicated on the left-hand side of Eq. (2), the parameter space of this BTHT has three dimensions: R/V , θ , and t_{cpa} . From Eq. (2), it is understood that the BTHT is a normalized integration along a bearing trace on the sonar B-scan. In other words, the BTHT is an averager that computes the arithmetic mean of the pixel value along the bearing trace. Assuming white Gaussian noise and one delay curve, an averager will provide a $5\log(N)$ dB processing gain. Hence, the BTHT will provide another $5\log(N)$ dB processing gain to the overall system in addition to the processing gain from the beamforming process. In reality, a passive sonar B-scan can exhibit more than one bearing trace. This theoretical process gain is difficult to achieve because of the overlapping bearing traces and interference clutter.

4. THE PASSIVE SHIPPING LANE MONITORING SYSTEM

Figure 3 is a conceptual diagram of the proposed passive shipping lane monitoring system. A sensor array of equally distanced hydrophones is laid parallel to a shipping lane of constant width. The interhydrophone distance is determined by the bandwidth of the signal of interest. The number of hydrophones is determined by the desired spatial resolution; the finer the required spatial resolution, the more hydrophones are needed. For each look direction, the sensor array uses beamforming techniques to integrate the acoustic energy from all of the hydrophones. For each observation instant, the integrated acoustic energies from various look directions are combined to form a row of the passive sonar B-scan. Although different beamforming techniques can be applied, the conventional plane-wave beamforming technique with Hanning shading is used in this report.

In the shipping lane, targets are assumed to travel parallel to the sensor array. They move either from left to right or from right to left. In Fig. 3, the arrow on the ship indicates its heading. Although targets need not travel parallel to the sensor array, for this report, we limit our study to the situations in which they do. (The BTHT can handle a range of different target headings, but this will increase the amount of

computation required.) This physical constraint limits the course (i.e., θ) of the target to be either 0° or 180° and reduces the dimensionality of the problem by one, reducing Eq. (1) to

$$B(t) = \left| \text{atan2} \left(\frac{R}{V}, (t - t_{\text{cpa}}) \right) \right| \text{ for } \theta = 0^\circ,$$

and

$$B(t) = \left| \text{atan2} \left(-\frac{R}{V}, -(t - t_{\text{cpa}}) \right) \right| \text{ for } \theta = 180^\circ.$$

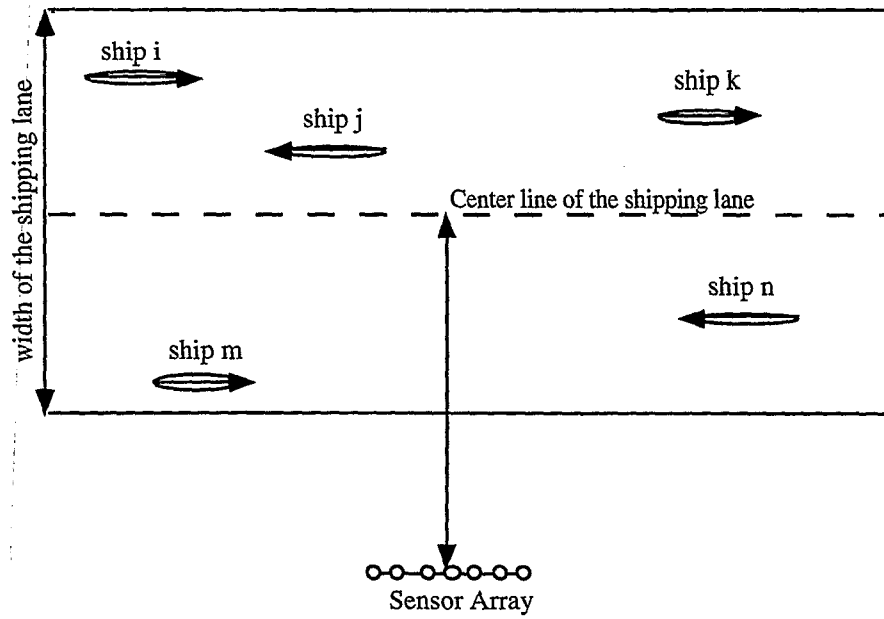


Fig. 3 — Proposed shipping lane monitoring system

Figure 4 is a data flow diagram of the BTHT. The sonar B-scan (i.e., the narrowband beamformer output) is input to the proposed BTHT. Since the target course is fixed at 0° or 180° , two 2-D BTHTs (one for 0° and one for 180°) are performed over the CPA time and V/R ratio (in this implementation, the V/R ratio is used instead of the R/V ratio to provide a better display of the effect of V on the output). Thus, two slices of the parameter space are generated. The parameter space (i.e., Hough space) for this problem can be viewed as two 2-D planes, each plane consisting of those cells that have the same target course but different V/R 's and t_{cpa} 's. The horizontal axis is the V/R -axis and the vertical axis is the t_{cpa} -axis. The existence of a peak in this two-dimensional plane indicates a target and its location represents the estimated target track parameters (V/R , t_{cpa}).

As mentioned earlier, many different sets of R and V can generate the same bearing trace as long as the V/R ratio is the same. Without additional information, this ambiguity cannot be resolved from a single sonar B-scan. Nonetheless, the speed of the target may sometimes be deduced from other acoustic

information, or historical information may confine V and R to a range of values that allows a proper choice of interval for R and V .

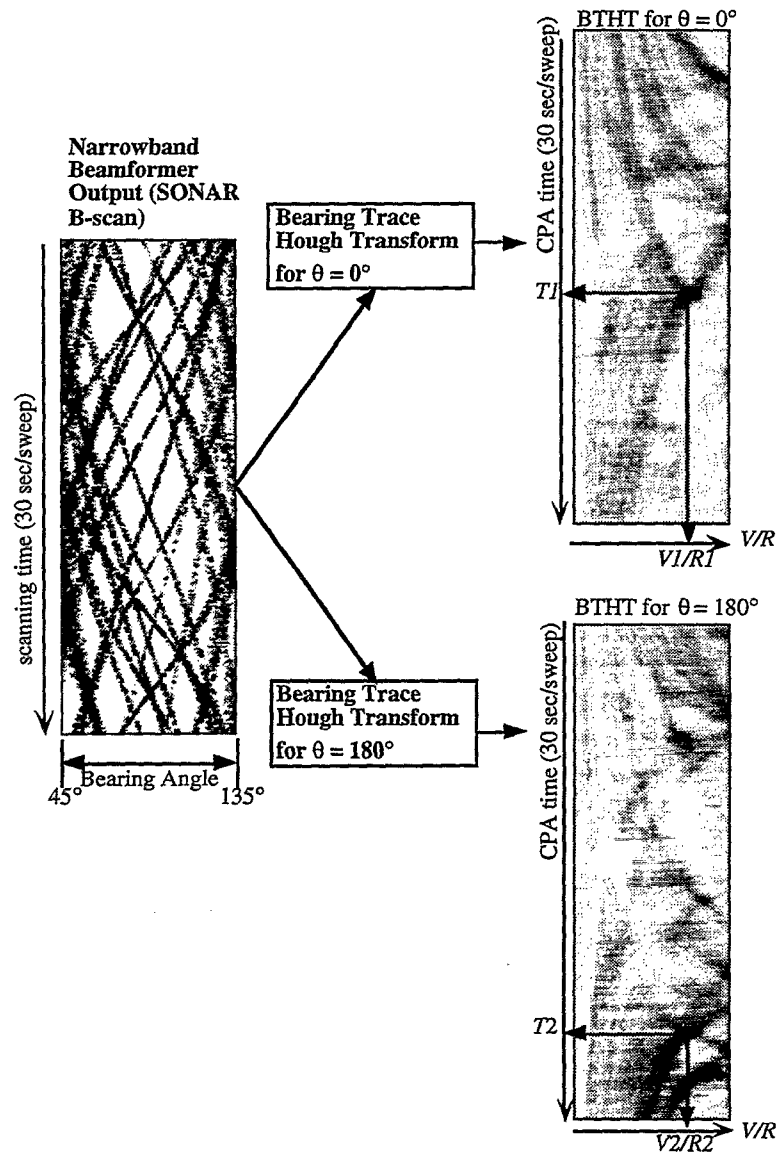


Fig. 4 — The narrowband sonar B-scan is processed using the 2-D Bearing Trace Hough Transforms for $\theta = 0^\circ$ and $\theta = 180^\circ$. The indicated peaks in the $\theta = 0^\circ$ and $\theta = 180^\circ$ Hough space show the existence of a target with a CPA at time $T1$ and a V/R ratio of $V1/R1$ and a target with a CPA at time $T2$ and a V/R ratio of $V2/R2$.

5. THE BTHT RESPONSE

To show the response of the BTHT, an ideal sonar B-scan for a scenario with seven targets moving parallel to the sensor array (Fig. 5) was generated using the arctangent equation of the bearing trace (Eq. (1)). All targets are assumed to be 2.7 nmi from the sensor array at the CPA and assumed to travel

parallel to the sensor array baseline with speeds between 3 and 13 kt. There are 256 sweeps, each sweep equivalent to 30 seconds of observation time, and 180 bearings (from 0° to 180° with a one-degree step). Table 1 lists the track parameters of the seven targets.

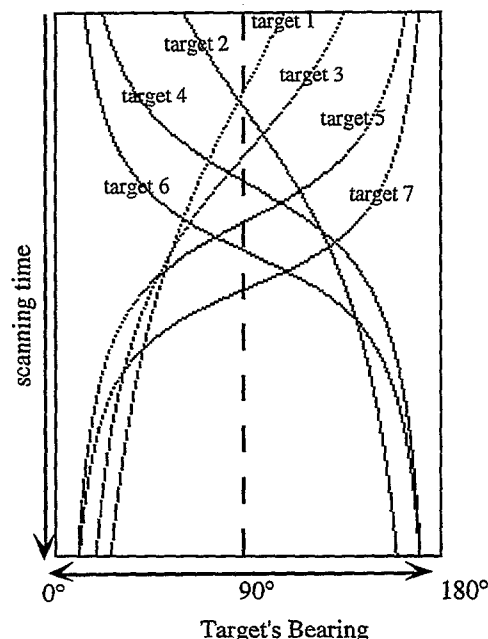


Fig. 5 — The ideal sonar B-scan for targets listed in Table 1. Note that targets 1, 3, 5, and 7 travel from left to right, while targets 2, 4, and 6 travel from right to left.

Table 1 — Track Parameters for Fig. 5

Target ID	Course (deg)	Speed (kt)	CPA Time (sweep)
1	0	3.0	32
2	180	4.0	48
3	0	5.0	64
4	180	10.0	80
5	0	11.0	96
6	180	12.0	112
7	0	13.0	128

This ideal sonar B-scan is processed by the BTHT, whose response is shown in Fig. 6. Figure 6(a) shows the output of the BTHT corresponding to $\theta = 0^\circ$. The four different peaks correspond to the four known targets with speeds of 3, 5, 11, and 13 kt. Figure 6(b) shows similar results of the BTHT for a target course of 180° . The three different peaks correspond to the three known targets with speeds of 4, 10, and 12 kt. All peaks are located at the expected [V/R, CPA time] coordinate in the parameter space.

The vertical axis is the CPA time in sweep; there are 256 sweeps. The horizontal axis is V in linear scale; there are 80 pixels in horizontal axis; assuming $R = 5000$ m, V spanned from 0.25 kt to 20 kt with a step $V_{\text{step}} = 0.25$ kt.

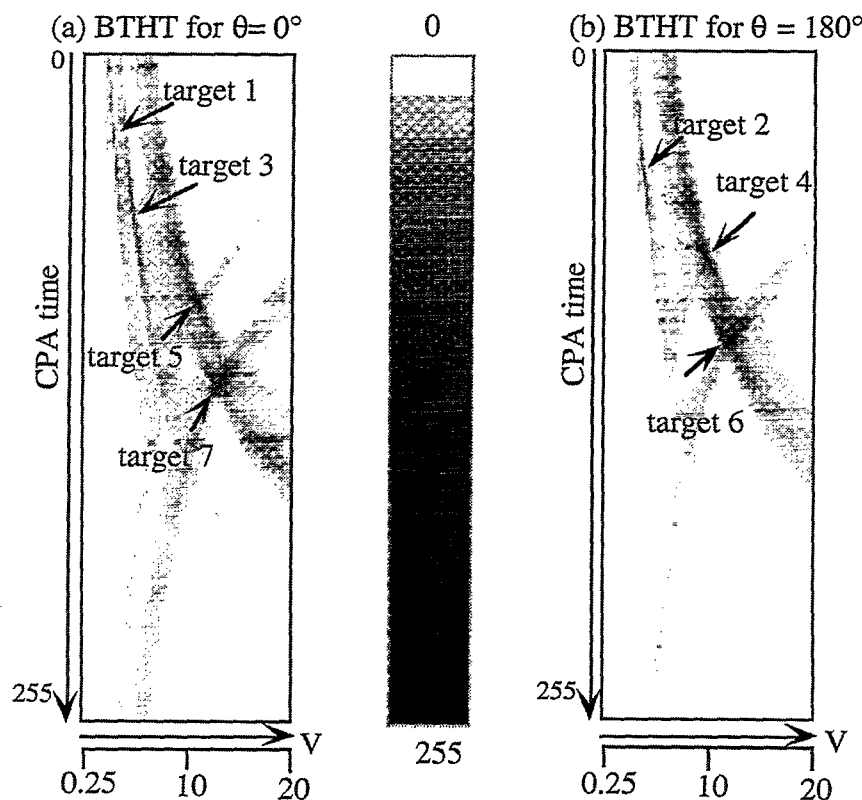


Fig. 6 — Response of the BTHT to the ideal sonar B-scan of Fig. 5, assuming (a) that the target moves from left to right, or (b) that it moves from right to left. V is given in knots.

6. EXPERIMENTS WITH SIMULATED ACOUSTIC DATA

To test the effectiveness of the BTHT in a real-world acoustic propagation environment, an acoustically simulated sonar B-scan (i.e., the one shown in Fig. 1) was used. The acoustic data were generated by a normal mode acoustic propagation model. The underwater environment is downward-refracting. The environmental change in the water column is assumed to be a function of depth only — it is range- and bearing-independent. As shown in Fig. 7, the water depth is 380 m and the bottom consists of a layer of sediment 50 m thick on top of a rock layer.

Figure 7 shows the sound velocity profile. The sound velocity profile is downward-refracting and range-independent. On the sea surface, the sound speed is 1537 m/s. In the water column, the sound speed varies between 1510 and 1537 m/s. In the sediment, the sound speed increases nonlinearly from 1681 m/s to 1780 m/s as a function of sediment depth. In the rocky layer, the sound speed varies from 1780 m/s to 1858 m/s. The acoustic data generated by the normal mode propagation model are then processed by a conventional plane-wave beamformer to produce the sonar B-scan for a signal frequency of 300 Hz. There are 200 ships generated in the simulation with various source levels and with CPAs ranging

between 5.94 and 21.06 nmi, speeds between 5 and 20 kt, depth between 2 and 5 m, and noise levels between 140 and 150 dB at 300 Hz.

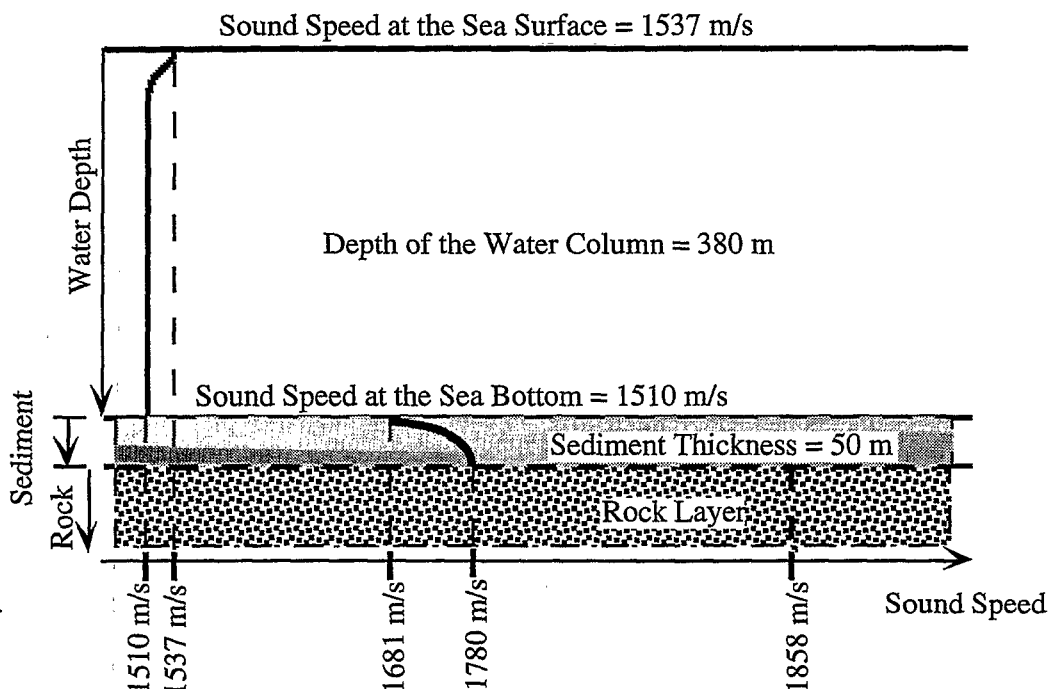


Fig. 7 — The sound velocity profile is downward-refracting and range-independent

The sensor array consists of 120 equally distanced hydrophones laid parallel to the shipping lane; the spacing between the hydrophones is 2.5 m. The width of the shipping lane is 28 km. The distance between the center line of the shipping lane and the sensor array is 25 km. Relative to the weakest signal source level, the background sea noise is modeled as 65 dB white noise while the loudest ship is at about 98 dB as received. Twelve targets are visible on the sonar B-scan.

Assuming that the target of interest will move at speeds between 0.25 kt and 20 kt and at a range of 13.5 nmi (25 km), with each sweep representing 30 seconds of observation time, the BTHT is performed over the V/R ranges from 1.54×10^{-4} to 1.23×10^{-2} with 80 different V/R's and 256 values of CPA time ($= 1$ sweep/cell). The 80 different V/R ratios are computed as $(V/R)_i = (i * V_{\text{step}} * 0.5144 * 30) / R$, where i ranges from 1 to 80; the value 0.5144 is the conversion factor from knots to m/s and the value of 30 is the duration of each sweep in seconds; and $R = 25000$ m. One BTHT is performed for an assumed target course of 0° and another is performed for 180° . To avoid the undesirable broad-beamwidth in the endfires (i.e., bearings near 0° or 180°) of the sensor array, the BTHT processes only the data from bearings ranging from 45° to 135° . Figure 8 shows the results of applying the BTHTs.

Figures 8(a) and (b) each show six different peaks corresponding to six different targets; they correspond to the 12 visible targets in the simulation. Note that the background, the intensity of the peak, and the sidelobes of the peaks are quite different from those of Fig. 6. For example, in Fig. 8, the peaks have a wider spread than those in Fig. 6. Some of the peaks have low intensity, barely above the

background. There are several reasons for this behavior. In the ideal sonar B-scan, for instance, the width of the bearing trace is one pixel, while the width of the bearing trace generated by the acoustic propagation model is larger than one pixel. Also, due to acoustic propagation, the bearing trace may not fit the arctangent function perfectly. This effect will reduce the intensity of the peak. The background sea noise and other weak targets also contribute values to the peak and background of the BTHT outputs. The overall effects are an increasing in intensity of sidelobes and the broadening of peak spreading. Tables 2 and 3 list the track parameters of the detected targets assuming a course of either 0° or 180° , respectively. The track parameters listed in Table 2 correspond to the BTHT output shown in Fig. 8(a), while those listed in Table 3 correspond to the BTHT output shown in Fig. 8(b). The resolution of V listed in Tables 2 and 3 is 0.25 kt. Note that the CPA time and the value of V/R are estimated from the location of the peak. The CPA ranges are derived based on a priori knowledge of the speed.

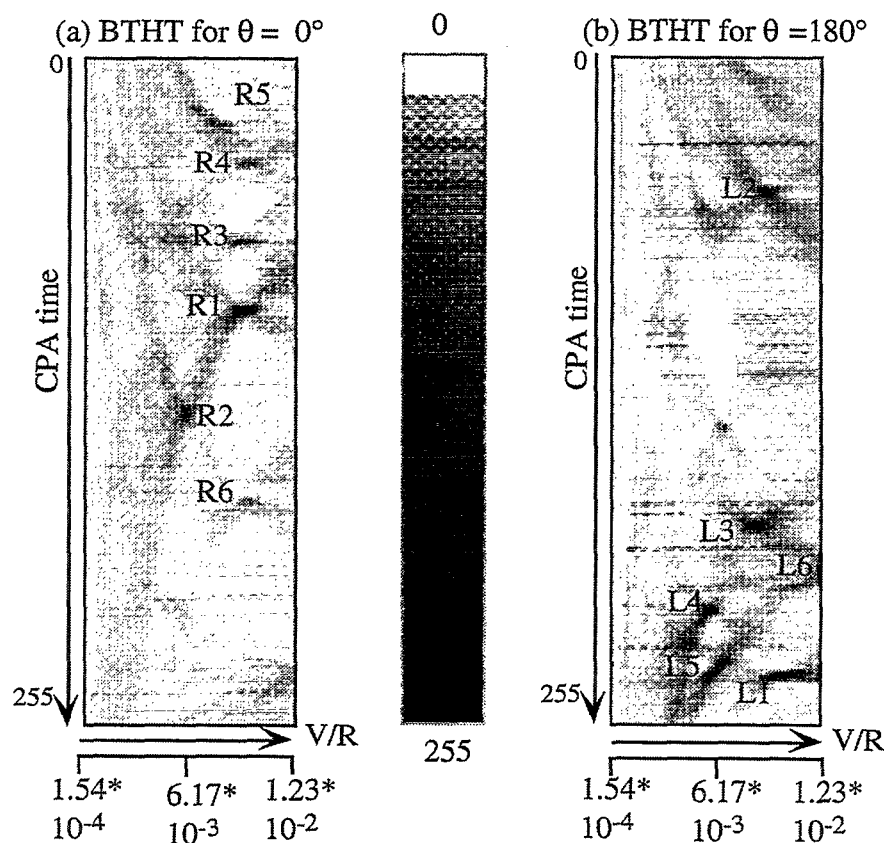


Fig. 8 — Results of the BTHT for the sonar B-scan shown in Fig. 1. In (a), the BTHT assumes that the target travels from left to right, while in (b), it assumes that the target travels from right to left.

Table 2 — Track Parameter of Detected Peaks with Assumed Course = 0°

Target ID	CPA sweep	V/R	V (kt)	R (km)
R1	97	0.0093	10.00	17
R2	136	0.0059	11.00	29
R3	70	0.0088	12.25	21
R4	40	0.0093	12.75	20
R5	25	0.0076	13.00	13
R6	169	0.0093	14.25	24

Table 3 — Track Parameter of Detected Peaks with Assumed Course = 180°

Target ID	CPA sweep	V/R	V (kt)	R (km)
L1	236	0.0102	10.00	15
L2	52	0.0088	11.50	20
L3	179	0.0083	12.25	23
L4	211	0.0057	13.0	35
L5	235	0.0060	14.25	36
L6	195	0.0120	15.00	19

7. CONCLUSIONS AND DISCUSSION

We have shown that the bearing trace on the sonar B-scan for a target traveling with constant speed and course is an arctangent curve. This allows us to formulate the problem of target detection and track parameter estimation as one of detecting an analytic curve using the Bearing Trace Hough Transform (BTHT) on the sonar B-scan. We tested the algorithm with a realistic acoustically simulated sonar B-scan to demonstrate its potential. In this report, the experimental results are based on single CPA range and two target headings, however, this situation is unlikely to occur in the real world. To handle multiple CPA ranges and multiple target headings, a search in high-dimensional Hough space (V , R , θ and t_{cpa}) is necessary. Similar to the Delay Curve Hough Transform [10,11], the onion-peeling process can also be applied to the BTHT for a better search of peaks in the high-dimensional Hough space. Lee and Shyu [12] proposed a nonlinear version of the Hough transform to control the sidelobe of peaks in Hough space. This is an interesting idea although it has not yet been tested with real data.

Several issues with respect to a fully automated deployable system remain to be resolved. They include the following:

- 1) What resolution should be used in the discrete parameter space? If the resolution of the searched parameter space is not fine enough, peaks may be missed or the parameter estimation may be inaccurate. On the other hand, increased resolution requires increased calculation.
- 2) What is the optimal integration time? Since the BTHT performs integration over the observation time, the longer the integration time, the higher the output signal-to-noise ratio. On the other hand, the target may not hold a constant speed over the entire integration time, which can degrade the peak values.

- 3) What target detection threshold should be set? If the threshold is set too high, the weak targets will not be detected; lowering the threshold, however, will increase the false alarm rate.
- 4) How many bearing values should be processed given that in a shipping lane, there will be some divergence of course from 0° or 180° ?

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